# Estimating Census Undercount using Coverage Measurement Survey and Demographic Data

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#### Abstract

Demographic analysis of data on births, deaths, and migration and coverage measurement surveys that use capture-recapture methods have both been used to assess US Census counts. These approaches have established that unadjusted Census counts are seriously flawed for groups such as young and middle-aged African-American men. There is considerable interest in methods that combine information from the Census, coverage measurement surveys and demographic information to improve Census estimates of the population. This article describes a number of models that have been proposed to accomplish this synthesis when the demographic information is in the form of sex ratios stratified by age and race. A key difficulty is that methods for combining information require modeling assumptions that are difficult to assess based on fit to the data. We propose some general principles for aiding the choice among alternative models. We then pick a particular model based on these principles, and imbed it within a more comprehensive Bayesian model for counts in poststrata of the population. The model is applied to data for African-Americans aged 30-49 from the 1990 Census, and results compared with those from existing methods.

Keywords: Post-enumeration survey; model selection; Gibbs sampling; posterior predictive distribution.

## 1. Introduction

Capture-recapture methods (Seber 1982) and demographic analysis (DA) of data on births, deaths, and migration have been used to estimate the undercount in the US Census (Robinson et al. 1993). These approaches have established that unadjusted Census counts are seriously flawed for groups such as young and middle-aged African-American men. Demographic analysis indicates a 1990 male-female ratio among African-Americans 30-40 of 0.91, whereas the published 1990 Census counts indicate a ratio of 0.86; with imputations and erroneous enumerations removed, the 1990 Census identified only 0.78 males for every female in this age-race category (Bell et al. 1996). Considerable effort has been devoted during the past decade to develop methods that combine demographic information and capture-recapture analysis to improve Census estimates of the population (Isaki and Schultz 1986; Fay, Passel and Robinson 1988; Choi, Steel, and Skinner 1988; Das Gupta and Robinson 1990; Wolter 1990; Bell 1993; Bell et al. 1996). Two key difficulties arise: DA typically provides estimates only for national levels of aggregation, and capture-recapture methods require modeling assumptions that are difficult to assess based on fit to the data. In the remainder of this section we overview existing research on these problems. Section 2 suggests principles for choosing among capture-recapture models, and Section 3 imbeds these methods within a unified Bayesian model. Section 4 applies the results of Sections 2 and 3 to the US Census data for African-Americans 30-49. A more complete description of our methods and results is available in Elliott and Little (1998).

#### Coverage Measurement Surveys

Since 1970, the Census has been supplemented by a coverage measurement survey (CMS), a detailed independent enumeration of households in a probability sample of Census blocks conducted immediately after the actual Census. In 1990, this CMS, termed the Post Enumeration Survey or PES, was conducted in July-September 1990, following the April-June 1990 data collection for the Census. To combine the Census and CMS data, imputations and erroneous enumerations were removed from the Census count, while persons in the CMS were cross-checked against Census records within the sample Census blocks and a set of surrounding blocks and assigned to either an In-Census or Out-Census category. After inflating the counts

Table 1: Observed data and associated underlying population parameters from Census and Coverage Measurement Survey:  $y_{k11}^S$  and  $y_{k01}^S$  are estimated counts of individuals in and out of the Census on the basis of the CMS follow-up;  $z_{k1}^S$  is the Census count, minus imputations and an estimate of erroneous enumerations;  $\psi_{kij}^S$  is the population that would reside in the ijth cell if the CMS had been a complete census. (stratum=k; gender=S).

Observed Data						Underlying Parameters		
		CMS				CMS		
		${ m In}$	Out			In	Out	
Census	$\operatorname{In}$	$y_{k11}^S$		$z_{k1.}^S$		$\psi_{k11}^S$	$\psi_{k10}^S$	$\psi_{k1.}^S$
	Out	$y_{k01}^{S}$				$\psi_{k01}^{S}$	$\psi_{k00}^{S}$	$\psi_{k0}^{S}$
		$y_{k,1}^S$				$\psi_{k.1}^S$	$\psi_{k.0}^S$	$\psi_{k}^{S}$

in the sampled blocks by the inverse of their probability of selection, a  $2 \times 2$  table (In-Out Census; In-Out CMS) was formed for males and females (S = M, F) in each of K post-strata, typically defined by geographic area and owner vs. renter status within age-by-race groupings (see Table 1). Table 1 also gives the "true" but unknown population counts in the kth poststratum for gender S that would be obtained if the CMS had itself been a census and if those missed in both the Census and CMS were known. These population quantities  $\psi_k = \{\psi_{ijk}^S : i = 0, 1; j = 0, 1; S = M, F\}$ , where i is the Census enumeration status and j the CMS enumeration status in the kth poststratum (1 if included, 0 otherwise), are considered unknown parameters, and are estimated by the statistical methods to be described.

### Model constraints

A fundamental problem (Bell 1993) is that Table 1 provides only 3 data elements to estimate the 4 parameters. Thus constraints must be placed on the parameters to obtain unique parameter estimates. One such constraint is to assume independence of capture and recapture (ICR), that is that the odds ratios for enumeration in the Census and the CMS for sex S in poststratum k,  $\theta_k^S = \frac{\psi_{k11}^S/\psi_{k10}^S}{\psi_{k01}^S/\psi_{k00}^S}$ , are all equal to 1 (Sekar and Deming 1949).

ICR: 
$$\theta_k^S = 1$$
 for all  $k, S$ . (1)

As Sekar and Deming point out, the ICR assumption can be violated when either the probabilities of capture and recapture are unequal or when they differ across individuals (unobserved heterogeneity), leading to "correlation bias". Correlation bias tends to lead to an underestimate the undercount, since if it is due to unobserved heterogeneity then  $\theta_k > 1$ , leading to an underestimation of  $\psi_{k00}^S$ . This form of bias is indicated by a pattern of implausibly low values of the male-female sex ratio, a value considered highly reliable by demographers (Robinson 1995), when the 1990 Census estimates are adjusted using the ICR model.

To correct for this bias, several models have been suggested that attribute the low observed male-female ratio to an undercount of males rather than an overcount of females. The estimated number of males in the population is increased to match the overall male-female ratio ( $\rho = \psi_{...}^M/\psi_{...}^F$ ) from DA. The additional males are distributed over the post-strata using a method based on the assumptions of the model. In practice  $\rho$  is estimated from DA within an age-race group, so the models are applied separately to each age-race grouping. In particular, Wolter (1990) assumed that the odds ratios for enumeration in the Census and the CMS are constant for males across the strata and constant and equal to 1 for females. We call this the fixed odds-ratio (FOR) model:

FOR: 
$$\theta_k^M = \theta^M$$
 and  $\theta_k^F = 1$  for all  $k$ . (2)

Bell (1993) and Das Gupta (Bell et al. 1996) extend Wolter's approach to four alternative models, all of which assume independence of capture-recapture for females ( $\theta_k^F = 1$  for all k) and adjust fitted counts so that their sum across poststrata matches the sex ratio  $\rho$  from DA. These models all add a single parameter

to the ICR model; all are "saturated" and provide an equally good fit to the data. Thus it is difficult to choose among alternative models, although they can yield adjustments with non-trivial differences. Another problem posed by these models is the existence of negative estimates of persons included in the Census but missed in the CMS, obtained by subtracting those estimated to have been in both the CMS and Census from the Census total ( $\widehat{\psi_{k10}^S} = z_{k1.}^S - y_{k11}^S$ ). Also unclear is how to account for uncertainty in the DA sex ratios. We consider each of these problems below, beginning with the problem of choosing among models.

## 2. Principles for Choosing Between Models

To simplify and reduce the scope of the model selection problem, we propose 6 principles for guiding the selection of a model for combining CMS and DA information:

- 1. PLAUSIBILITY: The model should imply a plausible description of Census behavior.
  - Assessment of this issue requires expert opinion and careful exposition of the model assumptions.
- 2. FIT: The model should minimize contradiction with available data.
  - The "no adjustment" model (that is, doing nothing) clearly fails this test, since it ignores the sizable body of evidence of differential undercount across demographic groups. A number of alternative models (including those considered by Bell and Das Gupta) provide better fits and hence should be preferred under this principle.
- 3. PREDICTION: The model should provide plausible predictions of key unobserved quantities, e.g. undercount rates should be within limits deemed reasonable.
  - Models may yield implausible outlying predictors for certain cells. While a consistent pattern of
    unlikely predictions is evidence that the model is not appropriate, in isolated cases modifications
    that control the extent of adjustments might be considered. These may be achieved informally
    by ad-hoc adjustments, or more formally by a Bayesian analysis based on prior distributions that
    limit the size of the adjustments.
- 4. ICR INCLUSION: The model should include the ICR model, which assumes zero correlation bias within poststrata, as a particular case.
  - It is harder to defend this principle as necessary on scientific grounds, but given the widespread adoption of the ICR model for CMS problems, it seems reasonable to restrict attention to the class of models that include that model for a particular choice of parameters. It is also in keeping with statistical parsimony: without evidence of correlation bias we would accept the independence model.
- 5. STABILITY: If alternative competing models are not distinguished on the basis of 1-4, models that yield more stable estimates of key estimands should be favored over models that yield less stable estimates.
  - If little can be concluded about the relative biases of competing models, then a model that yields estimates with reduced variance is to be preferred.
- 6. CONSERVATISM: If alternative competing models are not distinguished on the basis of 1-5, then models that are more conservative with respect to undercount adjustment should be favored over models that are less conservative.
  - Given the sentiment against any type of adjustment in some quarters, the goal of adjusting the Census counts to the minimal extent needed for consistency with DA and CMS data seems appropriate.

Of the six models considered by Wolter (1990), Bell (1993), and Das Gupta (1996), the fixed relative-risk (FRR) model that assumes a constant relative risk for enumeration in the Census and CMS for males appears to best meet the criteria considered:

FRR: 
$$\gamma_k^M = \frac{\psi_{k11}^M/\psi_{k1}^M}{\psi_{k01}^M/\psi_{k0}^M} = \gamma^M \text{ for all } k.$$
 (3)

In particular, only it and the FOR model include the ICR model as a special case. In addition, the FRR model appears somewhat more stable than the FOR model (Bell 1993). Hence we highlight this model in the remainder of our paper, although this choice might be changed by more exhaustive analyses. Our conclusions appear consistent with the comments in Bell (1993), although he is less sanguine in terms of final model choice.

# 3. A Fixed Relative-Risk Model Incorporating CMS and DA Data

This section outlines a comprehensive model for the underlying 8K population counts in the CMS tables that incorporates information about sex ratios from DA and eases prior specifications. For the FRR model (3), we reparameterize the eight population counts in poststratum k,  $\psi_k = \{\psi_{ijk}^S : i = 0, 1; j = 0, 1; S = M, F\}$ , as  $\psi_k^* = (\psi_{k...}, \rho_k, \delta_k^M, \delta_k^F, \phi_k^M, \phi_k^F, \gamma_k^M, \gamma_k^F)$  where:

- 1.  $\psi_{k...}$ , the total population count in poststratum k.
- 2.  $\rho_k = \frac{\psi_{k...}^M}{\psi_{k...}^F}$ , the sex ratio  $(\psi_{k...}^M + \psi_{k...}^F = \psi_{k...})$
- 3.  $\delta_k^S = \frac{\psi_{k1}^S}{\psi_k^S}$ , the Census undercount proportion for sex S
- 4.  $\phi_k^S = \frac{\psi_{k_{11}}^S}{\psi_{k_{1}}^S}$ , the proportion of Census cases enumerated in the CMS for sex S
- 5.  $\gamma_k^S = \frac{\psi_{k11}^S/\psi_{k1}^S}{\psi_{k01}^S/\psi_{k0}^S}$ , the relative proportion of Census and non-Census cases enumerated in the CMS for sex S

The above parameterization is particularly useful for the FRR model; other choices of parameterizations are more natural for other models. We then select the following independent priors for each parameter:

- $p(\psi_{k..}) \propto 1$ , a flat prior corresponding to our lack of knowledge about the total population counts in each poststratum.
- $\rho_k \sim N(\rho, \sigma^2)$  subject to the constraint that  $\sum_k w_k \rho_k = \rho$  where  $w_k = \frac{\psi_k}{\sum_k \psi_k}$  and  $\rho$  is the DA-estimated nationwide sex ratio. Variation in the sex ratios across poststrata is modeled via the parameter  $\sigma^2$ . The normal distribution is chosen for computational convenience.
- $\delta_k^S \sim BETA(a^S, b^S)$  and  $\phi_k^S \sim BETA(c^S, d^S)$ , beta priors that smooth the proportion of Census undercounts and the proportion of Census cases enumerated in the CMS across poststrata independently for each sex and provide a support of [0,1] for these proportions.
- $\gamma_k^M = \gamma^M \sim GAMMA(\alpha, \beta)$  for all k;  $\gamma_k^F = 1$  for all k. These priors assume that the relative proportion of Census and non-Census cases enumerated in the CMS for each poststratum is a constant across poststrata (likely greater than 1) for all males and is constant and known to be equal to 1 (under the independence assumption) for females.

In addition, we assume that

- $y_{k11}^S \mid \psi_{k11}^S \sim N(\psi_{k11}^S, (\tau_{k11}^S)^2)$
- $y_{k01}^S \mid \psi_{k01}^S \sim N(\psi_{k01}^S, (\tau_{k01}^S)^2)$
- $z_{k_1}^S \mid \psi_{k_1}^S \sim N(\psi_{k_1}^S, (u_{k_1}^S)^2).$

where  $y_{k11}^S, y_{k01}^S$ , and  $z_{k11}^S$  are all independent.

The mode of the posterior distribution of  $\{\psi_k^*\}$  might be computed by a numerical optimizing algorithm. However, given the large number of parameters relative to the data and presence of peaks near the boundary of support for  $\phi_k^S$  in cells where  $z_{k1}^S < y_{k11}^S$ , Newton-Raphson and Fisher scoring algorithm performed poorly. Hence we used Gibbs sampling (Gelfand and Smith 1990; Gelman and Rubin 1992; Smith and Roberts 1993) to draw estimates of the population parameters from their joint posterior distribution. The weighted sum of within-stratum sex ratio estimates was constrained to equal "known" nationwide sex ratio via the SWEEP operator (Goodnight 1979; Little and Rubin 1987). Use of the Gibbs sampling approach estimates the entire posterior distribution and thus allows for greater flexibility in terms of point estimation and inference. We focus primarily on posterior means, easily estimated as the mean of the parameter draws after an initial "burn-in".

The variances  $(\tau_{k11}^S)^2$ ,  $(\tau_{k01}^S)^2$ , and  $(u_{k1}^S)^2$  are treated as known. The beta hyperparameters  $a^S, b^S, c^S, d^S$  are estimated using Gibbs sampling assuming a uniform hyperprior distribution. Since little information is available to estimate the gamma hyperparameters  $\alpha$  and  $\beta$ , we chose the "flattest" prior for which  $P(\gamma^M \in [0.5, 2.0]) = 0.95$ ; this yielded  $\alpha = 9$  and  $\beta = 0.1306$ . Rather than attempting to find an empirical Bayes estimate of  $\sigma^2$ , we set  $\sigma = 1$ , which essentially allows the poststratum sex ratios to vary freely, subject to the constraint that they yield the DA estimate when aggregated.

# 4. Application to US Census Data

We now apply the methods described above to the 1990 Census for African-Americans aged 30-49, stratified into 12 poststrata. Poststrata 1 through 6 include those residing in owner-occupied dwelling units in urban area 250,000/+ in the Northeast (1), South (2), Midwest (3), and West (4); owners in urban areas under 250,000 (5); and owners in non-urban areas (6). Poststrata 7 through 12 include those residing in non-owner (rental) dwelling units in the corresponding geographic areas. Five chains consisting of 1,000 draws of the Gibbs sampler were run from different starting points, with the first 200 discarded as an initial "burn-in". The Gelman-Rubin test of convergence (Gelman et al. 1995), which measures the ratio of the total posterior variance to the mean within-chain variance, indicated an acceptable degree of convergence (max  $\sqrt{\hat{R}} = 1.07$ ).

Figure 1 and Table 2 indicate the differences between the population estimates in each poststratum for African-Americans 30-49 using (a) Census estimates (minus imputations and estimates of erroneous enumerations); (b) maximum likelihood estimates for the ICR model; (c) maximum likelihood estimates for the FRR model adjusted to DA sex ratios using Bell's (1993) approach; and (d) posterior means under the model of Section 3. Note that estimates for (b) and (c) are identical for females.

Several key observations can be derived from Figure 1:

- The undercount appears to be greatest, as might be expected, in Poststrata 7 through 10 (renters residing in urban area of 250,000/+).
- The FRR models provide larger estimates than the ICR models because these models adjust for correlation bias by forcing the total male/female ratio to equal or approximate DA sex ratio estimates.

(Recall that females are assumed to have zero correlation bias.) This bias appears to be associated, again as one might expect, with the undercount itself, appearing larger in the rental poststrata than in the owner poststrata.

- The estimates derived for males from the Bayesian approach of Section 3 under the FRR assumption generally fall between the maximum likelihood estimates of Bell (1993) for the FRR model and the MLE ICR estimates. A discrepancy between the MLE and Bayesian FRR model appears in Poststratum 10 for men (non-owners in Western urban areas). This can be explained in part by the fact that this poststratum (a) has the largest proportion of males as estimated by the post-CMS-adjusted data (SR=1.030 under maximum likelihood for the ICR model and SR=1.170 under maximum likelihood for the FRR model), and (b) has apparently poor Census coverage as estimated by the CMS: this poststratum contained the smallest proportion of the CMS that were identified in the Census and third-smallest proportion of those in the Census who were estimated to have been in the CMS. However, this estimate of poor coverage is based on relatively unstable CMS estimates (the largest CV for  $y_{k11}^M$ ). Thus the Bayesian approach identifies and "corrects" to some degree this potential outlier, increasing its estimated coverage toward the all-strata mean. Similiar discrepancies in Stratum 6 and 7 result in part from large CVs from the CMS estimates that allow the posterior results to be pulled toward the ICR estimates.
- Female estimates under the Bayes FRR models are somewhat smaller on average than under the MLE FRR model, possibly a consequence of the smoothing of the sex ratios. Exceptions are Strata 4 and 11, where the removal of the large negative cells increases the estimate of the female population over the MLE estimates.

The total undercount for African-American women 30-49 when compared against the total Census estimate for African-Americans 30-49 is 1.7% under the ICR/MLE FRR model and 0.5% under the Bayesian FRR model. The undercount for African-American men 30-49 is estimated to be -1.1% under the ICR model, 6.7% under the FRR model using Bell's 1993 maximum likelihood approach, and 5.4% under the Bayes FRR model. The total undercount is 0.4% for the ICR model, 4.1% for FRR maximum likelihood, and 2.9% for FRR Bayes. Thus the smoothing has added an element of conservatism (see Principle 6 in Section 2) that we think is an advantage of our approach. Note that the undercount here and below is compared to the published US Census estimates for African-American women 30-49 (4,484,162) and men 30-49 (3,859,304) (US Census Bureau 1991), not to the adjusted estimates minus imputations and erroneous enumerations shown in Figure 1.

# Inference

Inferences about the posterior distribution of parameters of interest can also be easily obtained from the distribution of the Gibbs draws. For example, an estimate of the 95% posterior probability interval (the Bayesian equivalent of a confidence interval) can be obtained by noting the 100th and 3900th smallest of the 4000 draws from the posterior distribution. Table 3 gives the mean and 95% posterior probability intervals (PPIs) for the total population in each of the 12 poststrata for African-Americans 30-49 under the Bayes FRR model, together with the adjusted Census estimate and Bell's (1993) maximum likelihood estimates under the ICR and FRR models. The 95% PPI for the undercount for females is (-1.8% - 2.6%); for males is (3.2% - 7.5%); and overall is (0.6%-5.2%).

#### Model fit: Negative Out CMS-In Census Cell Values

If matches from the CMS were overstated, erroneous enumerations from the Census were overstated, or some other source of bias is present, then the negative Out CMS-In Census cell values obtained by subtracting the In CMS-In Census population estimate from the adjusted Census estimates may be due to bias rather than sampling variance. Hence the negative Out CMS-In Census counts potentially provide evidence of lack of

Figure 1: Total Estimated Population by Gender and Stratum Under Census and ICR, FRR MLE, and FRR Bayes Models.

Table 2: Estimates of total population within each poststratum (in thousands): Census; ICR MLE; FRR MLE; Bayes FRR Posterior Mean and 95% Posterior Probability Interval under Bayes FRR with unconstrained SR. PS = Poststratum

strained 5h. r5 — roststratum												
	Census	ICR	FRR	Bayes	Bayes FRR							
PS	Estimate	MLE	MLE	FRR	95% PPI							
1	455	515	527	523	(498 - 550)							
2	1008	1097	1120	1109	(1079 - 1142)							
3	568	623	637	629	(610 - 650)							
4	260	304	314	326	(278 - 384)							
5	677	734	750	748	(716 - 781)							
6	598	687	708	676	(614 - 739)							
7	751	1001	1058	1015	(922 - 1128)							
8	1005	1221	1282	1278	(1191 - 1381)							
9	562	692	728	725	(682 - 774)							
10	373	468	500	479	(419 - 555)							
11	714	820	851	863	(821 - 909)							
12	186	216	224	214	(192 - 242)							

Figure 2: Posterior Predictive Draws of Census Estimates Minus In Census-In PES Estimates: Females (Data given by Tick Marks).

fit of the data to the model. To examine this possibility, we utilize posterior predictive distributions (PPD) (Gelman et al. 1995).

Classic p-values represent the probability under the model that the observed statistics  $T(y;\theta=\hat{\theta})$  will be less than (or greater than) the values of the statistic that would be seen in repeated observations:  $P(T(y) \leq T(y^{rep}) \mid \hat{\theta})$ . The PPD p-value represents the probability that the observed statistic  $T(y,\theta)$  is more extreme than replicated statistic, conditional on the observed data:  $P(T(y,\theta) \leq T(y^{rep},\theta) \mid y)$ . PPD p-values can be obtained from the draws of  $\theta$  generated by the Gibbs sampler;  $y^{rep}$  can be drawn from  $f(y \mid \theta^{rep})$ , and  $T(y^{rep}, \theta^{rep})$  compared with  $T(y, \theta^{rep})$ . Examining the histograms for females in Figure 2 shows that the only stratum for which the observed data appear in the tail of the predictive distributions is poststratum 4; the PPD p-value is 0.054. The smallest PPD p-value for males is 0.070, also in stratum 4. Overall then, the model fit is reasonably adequate for African-Americans 30-49, although there is modest evidence (made more modest by the number of comparisons) that the negative cells in Stratum 4 may be due to some form of bias unaccounted for in the model, either because of overestimation of the proportion of CMS subjects who were also captured in the Census or of overestimation of erroneous enumerations and imputations in the Census data.

#### 5. Discussion

In this paper we summarize methods proposed for incorporating post-Census CMS and demographic data into estimates of Census subpopulation counts. All methods face the difficulty that the underlying cell counts in the  $2 \times 2$  Census-CMS poststratification tables are unidentifiable unless a model is posited for the population. The simplifying assumption of independence – that the probabilities of capture and recapture are independent and homogeneous across the population – leads to ratios of males to females that are typically lower than estimates from demographic analysis. Numerous plausible models can be suggested that incorporate this

sex ratio data, all providing perfect fits to the data. We have suggested six principles – PLAUSIBILITY, FIT, PREDICTION, INDEPENDENCE MODEL INCLUSION, STABILITY, and CONSERVATISM – to help choose among the models. Use of these principles suggests a fixed relative-risk for enumeration in the CMS and Census (FRR) model (see (3)) as a leading candidate model for selection.

Beyond these qualitative discussions, we have described a more comprehensive statistical model that, through judicious choice of parameterization and prior distributions, eliminates negative cell estimates from the In Census-Out CMS cell of the poststratification tables and reduces outlying predictions of undercount rates. Applying this approach to the FRR model using 1990 Census data for African-Americans aged 30-49 yielded estimates of undercount of 0.5% for women, 5.4% for men, and 2.9% overall in this race-age category, with corresponding 95% posterior undercount intervals of (-1.8% - 2.6%), (3.2% - 7.5%), and (0.6%-5.2%). The estimates 1990 Census undercount among African-Americans aged 30-49 using the FRR MLEs (Bell 1993) are 1.7% for women, 6.7% for men, and 4.1% overall; no confidence intervals are easily available. Our approach identifies potential outliers in the poststrata tables and reduces their impact on post-CMS total population estimates. This also has the effect of adjusting to the minimum extent necessary to be consistent with the data, consistent with our CONSERVATISM principle. Use of posterior predictive distributions also indicates that the large negative raw In Census-Out CMS cells are consistent with variance in CMS enumeration or Census estimates.

Many extensions of the methods and models described could be envisaged. One immediate extension would be to introduce a prior for  $\rho$  to account for known uncertainty in the DA-estimated nationwide sex ratio. Also, the estimates of variability in the CMS and Census data are treated as fixed; prior distributions could be assumed to estimate any uncertainty in their values. Additional demographic measures could be incorporated in our model through careful choice of parameterizations. Prior means for the poststrata cell data or sex ratios could be regressed on poststratum characteristics to further reduce the dimensionality of the model. Alternatives could be examined to the underlying assumption that deviations from the ICR model are confined to males.

One criticism of our proposed approach is that it is explicitly Bayesian, and hence incorporates subjective elements through the choices of model and prior. However, every method of modern Census enumeration aimed at getting counts of the full population requires subjective assumptions – including methods that leave raw Census counts unadjusted. The Bayesian framework makes these assumptions explicit and open to debate, rather than implicit in the estimation algorithm. A second criticism is that the computations are complex, and simple transparent methods that are relatively easy to explain to lay audiences are preferable. We too favor simplicity, but think a distinction needs to be made between the underlying assumptions of the model, which are not particularly complex and are capable of being transmitted in non-technical terms, and the algorithms used to simulate posterior distributions based on the model, which are very complex but need not be understood by non-statistical stakeholders. More generally, complex micro-simulation and statistical models with subjective assumptions underlie the interpretations of much data that are used to inform public policy in the economic and health arenas.

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